Estimation of Velocity of a Frictionless Motion of a Truck on an Infinitely Long Straight Rail

Ogwola, P., and Sullayman, M.B.
Department of Mathematics,
Nasarawa State University, Keffi, Nigeria
ogwolapeter10@gmail.com

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Abstract
This paper is concerned with estimation of velocity of a frictionless motion of a truck on an infinitely long straight rail. For simplicity assume that the Truck is controlled only by the throttle producing an accelerative force per unit mass. A discrete dynamic model of first order difference equation is to describe the system. Kalman filtering technique is applied to the discrete dynamic model to estimate the velocity of the Truck at any particular time. A computer programme is developed to simulate the system.

Keywords: Distance, estimation, kalman filter, modelling, velocity.

Introduction

In system analysis, a fundamental problem is to provide values for the unknown states or parameters of a system given noisy measurements which are some functions of these states and parameters.

According to Greg and Gary (2006), Kalman filter is defined as a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the square error. Kalman filter was originally developed for use in spacecraft navigation but turns out to be useful for many applications.

Kalman (1960) published the discrete-time filter in a Mechanical Engineering Journal and Kalman and Bucy (1961), the continuous-time filter. In the meantime, Swerling (1959) had derived an equivalent formulation of the Kalman filter and applied it to the problem of estimating the trajectories of satellites using ground-based sensor. His results were published in an Astronomy journal the year before Kalman (1960) appeared.

Materials and Methods

Kalman filter Wikipedia

The Kalman filter model assumes the true state at time k is evolved from the state at (k-1) as stated below.

$$X_{k+1} = \Phi X_k + BU_k + \xi_k \tag{1}$$

Where,

Φ is the state transition model which is applied to the previous state X_k;

B is the control-input model which is applied to the control vector U_k

ξ_k is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance Q.

$\xi_k \sim N(0,Q)$

At time k an observation (or measurement) Y_k of the true state X_k is made according to

$$Y_k = HX_k + \eta_k \tag{2}$$

The Kalman filter loop

The Kalman filter loop given below summarizes what is known as the Kalman filter.

Enter prior estimate $\hat{X}_{k|k-1}$ and its covariance $\mathbf{P}_{k|k-1}$

Where H is the observation model which maps the true state space into the observed space and η_k is the observation noise which is assumed to be zero mean Gaussian white noise with covariance R, $\eta_k \sim N(0,R)$

The initial state, and the noise vectors at each step $\{X_o, \eta_1, \ldots, \eta_k, \xi, \ldots \xi_k\}$ are all assumed to be mutually independent.

The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state and as such no history of observations and /or estimates is required. In what follows, the notation $\hat{x}_{k|k-1}$ which is the 1 step prediction represents the estimates of x_k at time k given observations up to and including at time k-1.

$$\hat{x}_{k|k-1} = \Phi \hat{x}_{k-1|k-1} + BU_k \tag{3}$$

The covariance matrix for the one step prediction error is given by

$$p_{k|k-1} = \Phi p_{k-1|k-1} \Phi^T + Q \tag{4}$$

$\hat{X}_{k|k-1}$ and $P_{k|k-1}$ are the Predicted (a priori) state estimate and Predicted (a priori) estimate of covariance respectively and represent the initial values for the Kalman filter.

The state of the filter is represented by two variables $\hat{X}_{k|k}$, the updated (a posterior) state estimate at time k given observations up to and including at time k and given by

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k(Y_k - H\hat{X}_{k|k-1}) \tag{5}$$

$P_{k|k}$, the updated (a posterior)error covariance matrix (variance of the estimation error) given by

$$P_{k|k} = (I - K_k H)P_{k|k-1} \tag{6}$$

Where K_k is the Kalman (Filter) gain and given by

$$K_k = P_{k|k-1}H^T(H P_{k|k-1}H^T + R)^{-1} \tag{7}$$

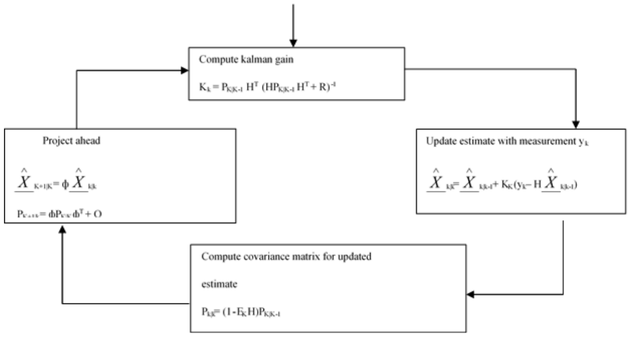

Figure 1: The Kalman Filter Loop as in Robert and Patrick (1992)

Once the loop is entered it can be continued for any N ($N \geq 1$) iterations, k = 0,1,……..,N-1.

Modelling and identification

Suppose a Truck is being driven along a straight rail, and let its distance from initial point be $x_1(t)$ at time t. For simplicity assume that the Truck is controlled only by the throttle, producing an accelerating force per unit mass. Ignoring friction, wind resistance etc. It is required to find the velocity of the Truck at time t = 1,2,3 ..... seconds.

Let $x_1(t) = x(t)$ (position) (8)

$$x_2(t) = \dot{x}(t) \text{ (velocity)} \tag{9}$$

Application of Newtons principle of classical mechanics yields

$$m\frac{d^2x_1(t)}{dt^2} = u(t) \tag{10}$$

For a unit mass, equation (10) becomes

$$\frac{d^2x_1(t)}{dt^2} = \frac{u(t)}{m} = u^*(t) \tag{11}$$

From eqns. (8), (9) and (11) we have,

$$\left.\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= u^*(t) \end{aligned}\right\} \tag{12}$$

Writing eqn. (12) in matrix form gives

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u^*(t) \tag{13}$$

Where

$u(t)$ =accelerating force

$u^*(t)$ = accelerating force per unit mass

Equation (13) can be written as

$$X_{k+1} = \Phi X_k + BU_k + \xi_k \tag{14}$$

Where,

$\Phi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $X_k = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, ξ_k is the modelling noise.

$U_k = u^*(t) = (1 \times 1)$ control matrix.

The estimation problem

The estimation problem is stated as follows: Given,

$$\left.\begin{aligned} \mathbf{X}_{k+1} &= \Phi X_k + BU_k + C + \xi_k \\ Y &= HX_k + \eta_k \end{aligned}\right\} \tag{15}$$

From the observed values of $Y_0, Y_1, \ldots\ldots\ldots\ldots, Y_k$, find an estimate $\hat{X}_{k|k}$ of X_k which minimizes the expected loss.

Where:

Φ = (2×2) constant matrix obtained from the transition model

B = (2×1) control input matrix which is applied to the control vector $\underline{U}_k$.

$\underline{Y}$ =(1×1) output vector (vector measurement at time t_k), $\underline{Y}$ =y, it is the measured value of the distance covered at t_k

H = (1×2) constant matrix giving the ideal connection between the measurement and the State vector at time t_k

$\underline{X}_k$= (2×1) process state vector at time t_k , i.e.,$\underline{X}_k$ =$\underline{X}$ (t_k) = $\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$, x_1 and x_3 are the estimates of distance and velocity of the truck at time t_k respectiveiy.

$\underline{U}_k$ = (1×1) control vector, $\underline{U}_k = u^*(t)$,

$\underline{\eta}_k$ = (1×1) measurement error –assumed to be white noise sequence with known co-Variance R and having zero cross correlation with $\underline{\xi}_k$ sequence.

$\underline{\xi}_k$ = (2× 1) vector-assumed to be white noise sequence with known co-variance

Recursive processing of the noisy measurement (input) data

A computer programme is written to simulate the system.

An algorithm for the programming model as in ogwola (2017) is as follows

Given the initial values $\underline{\hat{X}}_{k|k-1}$ and its co-variance, $p_{k|k-1}$

(i) $K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R)^{-1}$

(ii) $\underline{\hat{X}}_{k|k} = \hat{X}_{k|k-1} + K_K(Y_k - H\hat{X}_{k|k-1})$

(iii) $P_{k|k} = P_{k|k-1} - K_k H P_{k|k-1}$

(iv) $\underline{\hat{X}}_{k+1|k} = \Phi \underline{\hat{X}}_{k|k}$

(v) $p_{k+1|k} = \Phi p_{k|k}\Phi^T + Q$

The values of $\underline{\hat{X}}_{0|-1}$ and $\mathbf{P}_{0|-1}$ used were $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.

$$\Phi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

The parameter Q, R that gives optimal estimates of velocity is $\begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$ and 0.4 respectively.

The distance (Y_k) of the truck at time t_k from the initial point is given in table 1.

Results

The values of Φ, Q,R, H , $\hat{X}_{0|-1}$,$P_{0|-1}$ and Y_k were used to run the computer programme from which the following results in table 2 were obtained.

Table 1: Observed (measured) values of the distance (Y_k) covered at time t_k of the Truck in Kilometres.

K	t_k	Y_k (Km)
0	t_0	4
1	t_1	7
2	t_2	9
3	t_3	15
4	t_4	16
5	t_5	20

Table 2: Simulation Results for the system

K	t_k	X_1	X_2
0	t_0	4.57	3
1	t_1	5.22	1.78
2	t_2	6.17	2.83
3	t_3	10.33	4.67
4	t_4	11.67	4.33
5	t_5	14.01	5.99

Discussions

In table 2, column 2 gives the time (t_k) it takes for the truck to reach a particular point away from the initial point. Column 3 gives the estimates of the true value of the distance (X_1) in km covered from the initial point at time t_k. Column 4 gives the estimates of the velocity(X_2) of the truck in km/s at any particular time

In conclusion, a sensor inside the truck measured the distance of the truck from the initial point at any time t_k. The value of the distance is then imputed into computer which estimate the true value of the distance (X_1) together with the estimates of the velocity(X_2) of the truck through the computer programme developed.

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