Wavelet Analysis of Stocks in the Nigerian Capital Market

[1]Okonkwo., C.U.,[2] Osu, B.O., [3] Uchendu, K., and [4]Chibuisi, C.
[#1]Department of Mathematics and Statistics Caritas University Amorji-Nike Enugu.
[2]Department of Mathematics Michael Okpara University of Agriculture Umudike.
[3]Department of Statistics, Michael Okpara University of Agriculture, Umudike, Nigeria.
[4]Department of Insurance, University of Jos, Jos, Nigeria

Abstract

In this paper, the relationship between some selected stocks in the Nigerian Capital Market was investigated using wavelet analysis. The selected stocks are Dangote Cement (Dans) representing the housing sector, Julius Berger (Jbger) representing the Construction industry, Nestle Nigerian Plc (Nese) representing the food and beverages sector and United Bank of Africa (Ubas) representing the banking sector. The goal is to find out how the different sector relate to each other, t also serve as a guide for investors in the Nigerian stock Market. The result shows that on low frequencies, the coherence between stocks are low but volatile. The cross wavelet showed that there is little or no co-variability in the magnitude of the movement among the selected stocks but there are co-variability in the direction of the movement. The continuous wavelet transform (wavelet spectrum) shows that for all the stocks, there is high volatility at the low frequency scales and low volatility at high frequency scales. The discrete wavelet transform (DWT) suggests that in the absence of noise, Nese (representing the food and beverage industry is the most volatile stock.

Keywords: wavelet analysis, wavelet coherence, stock price, Portfolio selection.

Introduction

The Nigerian Stock Exchange

The Nigerian Stock Exchange was established first as Lagos Stock Exchange in 1960 and later renamed the Nigerian Stock Exchange in 1977 with branches established in some major commercial cities. In 1961, when trading started, there were 19 securities listed for trading, as at 31st May 2018, there are 169 securities listed on the Exchange with a market capitalization of over 13 trillion NGN. The 2016 Index Mundi ranking of the world stock markets based on market capitalization placed Nigeria in the 56th position in the world and 3rd in Africa, trailing the South African and the Egyptian Stock Markets.

Investors in the stock market can be divided into two broad categories; the short term investors and the long horizon investor. Portfolio selection should be done in such a way as to achieve the investors' goal which is to grow his portfolio and to mitigate loss. This paper seeks to use the wavelet transform to provide a guide for investors and would be investors

Literature Review

Wavelet analysis has found increasing application in the physical science, medical sciences and engineering than in economics and finance. Of recent however, there is an increasing application of wavelet in the area of finance and economics. This is because of its ability to analyze information both in the time and frequency domain.

Some of the application of wavelets in economics and finance in recent times is on the rise. This can be seen in(Razdan, 2004)where continuous wavelet transform(CWT)was used to examine two highly correlated Indian financial time series Bombay stock exchange index (BSE) and National Stock exchange index (NSE

Wavelet methods provide a unified framework for measuring dependencies between variables overtime and frequencies. These methods allow the analysis of the relationship between stock markets and oil prices over many time scales. This approach provides a better understanding of the dynamic relationship between variables. (Gourène & Mendy, 2018)

The applications of the wavelet methods in this relationship are illustrated in the world's major stock markets in studies such as those of (Jammazi & Aloui, 2012), (Gourène & Mendy, 2018). António & Luis, (2009) did a wavelet analysis of the stock market of Germany, Japan, UK and the US. (Tiwari, Dar, Bhanja, & Shah, 2013) looked at the multiple wavelet analysis of nine Asian stock market.

Various work had been done on the Nigerian stock market volatility using the GARCH model (Kolade, 2013; Emenike and Eleke, 2012; Okonkwo *et al*, 2018).

Investors are usually risk averse, however the degree differ with different investors. Different investors have different motivation and levels in terms of their expectation of risk and return on their investment portfolio. The short term investors would be interested in the high frequency shocks (low time scale) as opposed to the long term investors who are more interested in the low frequency shocks (high time scale) to the market. It can be argued that from a portfolio diversification point of view, the short-term investor is naturally more interested in the co-movement of stock returns at higher frequencies, that is, short-term fluctuations, but the long-term investor focuses on the relationship at lower frequencies (long-term fluctuations).Diversification strategies performed by international investors also depend on the nature and magnitude of the existing relationships between different stock markets. Understanding the

Interrelations among the various markets is therefore important to diversify risk and to derive high returns.(Masih & Majid, 2013)

Materials and Methods

3.1 Wavelet Analysis

A wavelet is a real-valued square integrable function $\psi \in L^2(\mathbb{R})$satisfying the conditions:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\frac{(t-\tau)}{s}, \quad s \in \mathbb{R}^+,\ \tau \in \mathbb{R}. \quad (1)$$

In (1), s is called a scaling or dilation parameter while τ, is called thetranslation or position parameter. Wavelets have certain characteristics such as oscillatory that is$\int_{-\infty}^{\infty} \psi(x)dx = 0$.

It is also integrable that is

$\int_{-\infty}^{\infty} |\psi(x)|dx < \infty$.

It satisfies the admissibility condition.

Definition A wavelet $\psi \in L^2(\mathbb{R})$ is said to be admissible if its Fourier transform,

$F(x) = \int_{-\infty}^{\infty} \psi(u) e^{-i2\pi xu}\, du,$

Satisfy

$C_\psi = \int_{-\infty}^{\infty} \frac{|F(x)|^2}{x} dx$

where $0 < C_\psi < \infty$.

Wavelet can be seen as small wave that grows and decays in a finite period of time. Morletetal (1982).

Examples of wavelets are Haar wavelet which can be defined as collection of orthonormal wavelets. It sscaling function also called the father wavelet is defined as:

$$\phi_{Haar} = \begin{cases} 1, & 0 \le t < 1 \\ 0, & otherwise \end{cases}$$

while the mother wavelet function of Haar, ψ, is given by:

$$\psi_{Haar} = \begin{cases} 1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t < 1 \\ 0 & otherwise \end{cases}$$

Using the mother wavelet, we can get other wavelets, usually called the daughter wavelets with the help of (2)

$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k) \qquad (2)$$

where j *and* k are integers.

The Morlet wavelet is given as:

$$\psi(t) = e^{\frac{t^2}{2}}[cos\omega_0 t + jsin\omega_0 t]$$
$$= e^{-\frac{t^2}{2}} e^{j\omega_0} = e^{-\frac{t^2}{2} + j\omega_0}$$

The mexican wavelet is given as:

$$\psi(t) = \frac{1}{\sqrt{2\pi\sigma^3}} e^{-\frac{t^2}{2\sigma^2}(\frac{t^2}{\sigma^2} - 1)}.$$

3.2 Continuous Wavelet Transform (CWT)

The continuous wavelet transform (CWT) is given as:

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi(\frac{t-\tau}{s})\, dt. \qquad (2)$$

In (2) $W_x(\tau, s)$can be described as a projection of a chosen wavelet on a time series. We can recover our original data using (3)

$$x(t) = \frac{1}{C_\psi} \int_0^{\infty} \left[\int_{-\infty}^{\infty} W_x(\tau, s)\psi_{u,s}(t) d\tau\right] \frac{ds}{s^2}, \quad s \neq 0 \qquad (3)$$

3.3 Discrete Wavelet Transform (DWT)

Discrete wavelet transform (DWT) are translated and scaled in discrete steps. The DWT decomposes the signal into mutually orthogonal set of wavelets. The DWT

Is given by:

$$\psi_{j,k} = \frac{1}{\sqrt{s_0}} \psi \frac{t - k\tau_0 s_0}{s_0} \qquad (4)$$

In equation (4)m and k are integers whiles$s_0$is strictly greater than 1.

The scaling function and the wavelet function of DWT are defined as:

$$\varphi(2^j t) = \sum_{i=1}^{k} h_{j+1}(k)\varphi(2^{j+1} t - k) \quad (5)$$

$$\psi(2^j t) = \sum_{i=1}^{k} g_{j+1}(k)\varphi(2^{j+1} t - k) \quad (6)$$

And then, anytime series *f*(*t*) can be written as (7) which is a linear combination of (5) and (6)

$$f(t) = \sum_{i=1}^{k} \lambda_{j-1}(k)\varphi(2^{j-1} t - k) + \sum_{i=1}^{k} \gamma_{j-1}(k)\psi(2^{j-1} t - k) \qquad (7)$$

Wavelet Power Spectrum (WPS)

The local wavelet power spectrum is the absolute squared of the wavelet transform.

$$WPS_f(s, \tau) = |W_{f,\psi}(s, \tau)|^2 \qquad (8)$$

The WPS depicts and measures the local variance of a signal at various scales s by giving information on the relative power (energy) at certain time and scale (frequency). Hence variance decomposition with a good time localization of the time series is done under investigation,

Cross-Wavelet Transform (XWT) and Cross-Wavelet Power (XWP)

Given two time series, $f(t)$and$g(t)$, the cross-wavelet transform (XWT) of the series is given as

$$XWT = W_{f,\psi} W_{g,\psi}^* = W_{f,g} \qquad (9)$$

With W_f and W_g as the series wavelet transforms respectively, and the cross-wavelet power is defined as

$$XWP_{f,g} = |W_{f,g}(s,\tau)| \qquad (10)$$

Whiles the wavelet power spectrum (8) depicts the series local variance, the cross-wavelet power of two time series (10) characterize the local co-variance at each time and frequency.

Wavelet Coherency

Coherency is analogue of classical correlation. To identify both frequency-bands and time-intervals when two signals are related, Wavelet Coherency is used. Given two time series $f(t)$ and$g(t)$, their wavelet coherency is defined as:

$$R^2_{f,g}(s,\tau) = \frac{|S(s^{-1}W_{f,g}(s,\tau))|}{\sqrt{S(s^{-1}|W_f|^2) \times S(s^{-1}|W_g|^2)}},\ 0 \leq R_{f,g}(s,\tau) \leq 1, \qquad (11)$$

where S is a smoothing operator defined as $S(W) = S_{\text{scale}}(S_{\text{time}}(W(s)))$,whereS_{scale} denotes smoothing along the wavelet scale axis andS_{time}smoothing in time.(Grinsted, Moore, & Jevrejeva, 2004) and is dependent on the choice of mother wavelet.

Values of the wavelet coherence close to zero indicate weak correlation while values close to one are an evidence of strong correlation

Data

The data for this work are some selected stock price of the Nigerian Stock Market from 06/01/2015 to 15/02/2017that is 512 observations. They are Dangote Cement (Dans), Julius Berger (Jbger), Nestle Nigeria (Nese) and United Bank of Africa (Ubas). See Table 1. The return series are represented graphically in Figure 1. The data are first transformed into return series by:

$$r_t = \log\left(\frac{p_t}{p_{t-1}}\right),$$

where $p_t$is the price of the stock at time t.
Each selected stock is assumed to be a sectorial representation of the nation's economy. (See Table 1).

Result

All computation was carried out using Scilab Wavelet Toolbox (SWT) in Scilab 5.5.2 and Microfit. The system used was HP 630 with processor Intel R. 32 bit operating system. 4Gb RAM. The summary statistics Table 3 shows that Dans and Nese are negatively skewed while Jbger and Ubas are positively skewed. The kurtosis shows that all the stock returns are heavy tailed as expected with Dans being closest to normality.

The correlation matrix is shown in Table 2 and Figure 2 indicates that there is a positive correlation among all the stocks except for Julius Berger that is negatively correlated with Nestle Plc with a correlation coefficient of -0.0171. The strongest positive correlation is between Dangote Cement and United Bank of Africa with a correlation coefficient of 0.1736. The weakest positive correlation is between Nestle Plc. and Dangote Cement with a coefficient correlation of 0.0068. From the foregoing, an investor will be warned against investing in Dangote and UBA simultaneously. He will be advised to have Julius Berger and Nestle Plc in his portfolio.

The analysis of the wavelet coherence shows that the stock returns are generally correlated at high frequencies. There is high volatility at low frequencies. Nestle and UBA have co-movement at frequencies of 32 and 64 scales and time of 100 and 320. They have high coherence. Dangote and Julius Berger are highly coherent at 16 – 64 period and at time 110 – 460. Julius Berger and UBA have coherence at 32 to 64 scale frequency and 90 to 210 time period. Dangote and Nestle have coherence at 16 to 32 scale frequencies and at time 370 to 450. Dangote and UBA have high coherence at frequencies 16 to 32 and time 0 to 200. Julius Berger and Nestle have high coherence at frequencies of 64 to128 and at times 0 to 500.

The wavelet coherence result elaborates the correlation matrix. Dangote and UBA should not be held simultaneously. Julius Berger and UBA are the best combination for the long term investment. Dangote and UBA should not be held together especially in the long run.

The result of the cross wavelet power shows that there is a very low covariance among all the variable pairs as the uniformly blue colour depict. However, the phase plot shows some co-variability among the variables. For Dans and

Nese, we observe that in the 0 – 100 time scale, Dans lags Neseat the at 0 – 4 frequency scale, Dan leads Neseat 8 -16 and 16 – 32 frequency scale. At 100 – 200 time scale, Dan and Nese are in phase at frequency scale 0 – 4, in anti-phase at 4 – 8 and then Dan leads Nese for most of the frequency scale 32 – 64 and at 64 – 128. At the 200 -300 time scale, Dans and Nese are out of phase at 0 – 4 frequency scale then Dans lags Nese at 16 – 32, they are in phase at 32 – 64 and they are in anti-phase at 64 -128. At 300 – 400 time scale, there seems to be minimal movement. At 400 -500 Dans leads Nese at 0 - 4 and 32 – 64 frequency scale while they are in phase at 16 – 32.For Dans and Jbger, Dans lags Jbger for most of the time and frequency scale and for other time they seems to be in phase. At few occasions, Dans leads Jbger. For Dans and Ubas, Jbger and Nese, Jbger and Ubas as well as Nese and Ubas, it is also a lead-lag relationship between the variables, sometimes in phase and other times at anti-phase. It however can be observed that the most active movement in the time domain is the 0 – 300 and 400 – 500 time scale, while the 300 – 400 time seem to less active except for Dans and Jbger, where the upper half of the 300 – 400 was relatively active.

The Continuous wavelet transform (CWT) result Figure 5 shows that for the selected variables, the volatility is high at low frequencies (usually between 0 and 16) and low at high frequencies. The phase plot shows that the direction of the movement is highly unpredictable.

In the Discrete Wavelet Transform (DWT) result Figure 6 shows the first plot is the original signal labeled d1(-1), from d2(-2) to d5(-2) the details of the signal are lost as the level increases. The approximation of the signal is given by s5(-1).From the approximation, it can be seen that Nese has the highest volatility in the absence of noise.

Table 1 Some Selected Stock Price Return

Stock	**Abbreviation**	**Sector**
Dangote Cement	Dans	Housing
Julius Berger	Jbger	Construction
Nestle Nigeria Plc	Nese	Food and Beverages
United Bank of Africa	Ubas	Banking

Table 2 Matrix of Correlation of the Stock Return

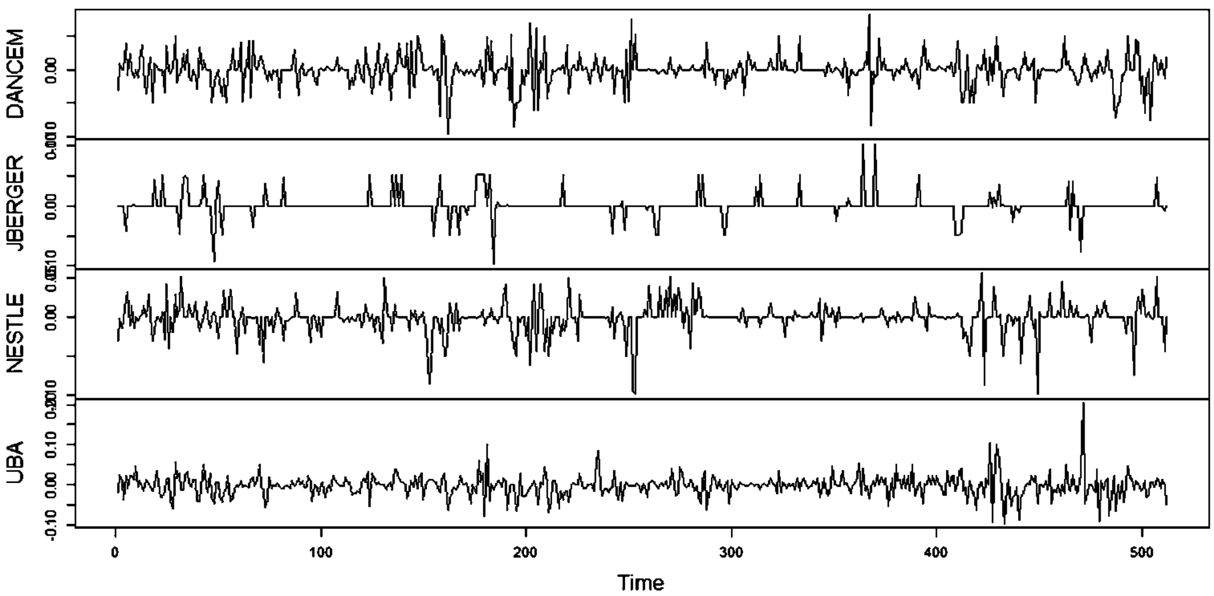
	Dans	**Jbger**	**Nese**	**Ubas**
Dans	1	0.0582	0.0068	0.1736
Jbger	0.0582	1	-0.0171	0.0394
Nese	0.0068	-0.0171	1	0.0855
Ubas	0.1736	0.0394	0.0855	1

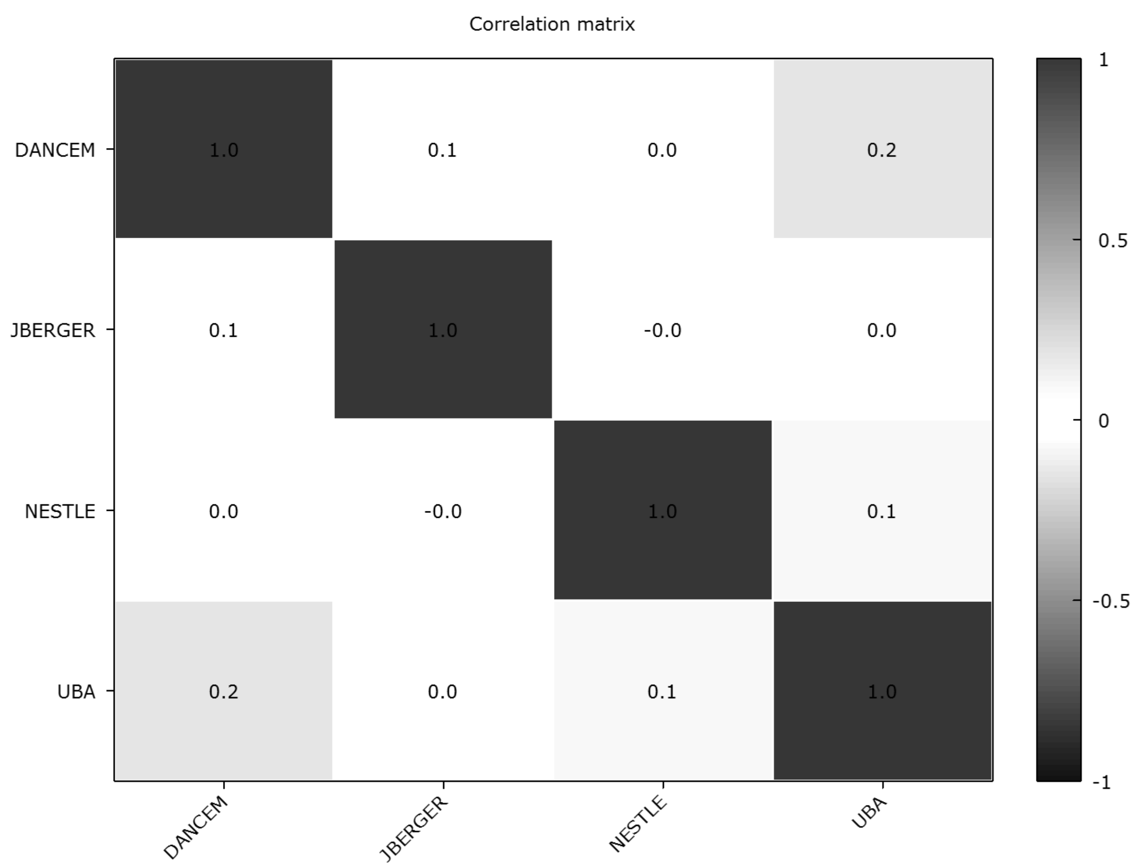
Table 3 Summary Statistics of the Stock Return

	Dans	**Jbger**	**Nese**	**Ubas**
Min.	-0.096	-0.097	-0.098	-0.097
Mean	-0.001	..00E-03	-0.001	-0.003
Max	0.082	0.102	0.056	0.204
Skew	-0.316	0.379	-1.218	0.729
Kurt	2.767	10.56	6.637	7.863

Conclusion

In this paper, the wavelet analysis of four stocks in the Nigerian stock market was examined. The result showed wavelet coherence are in agreement with the correlation matrix implying that the wavelet coherence is a good measure of correlation among stock price return at least in the time domain. However the wavelet coherence gave more details than can be seen in the correlation matrix. The cross wavelet showed that there is little or no co-variability in the magnitude of the movement among the selected stocks but there are co-variability in the direction of the movement. The continuous wavelet transform (wavelet spectrum) shows that for all the stocks, there is high volatility at the low frequency scales 0 – 4, 4 – 8, and 8 – 16. There is low volatility at high frequency scales. Finally the discrete wavelet transform (DWT) suggests that in the absence of noise, Nese (representing the food and beverage industry is the most volatile stock.





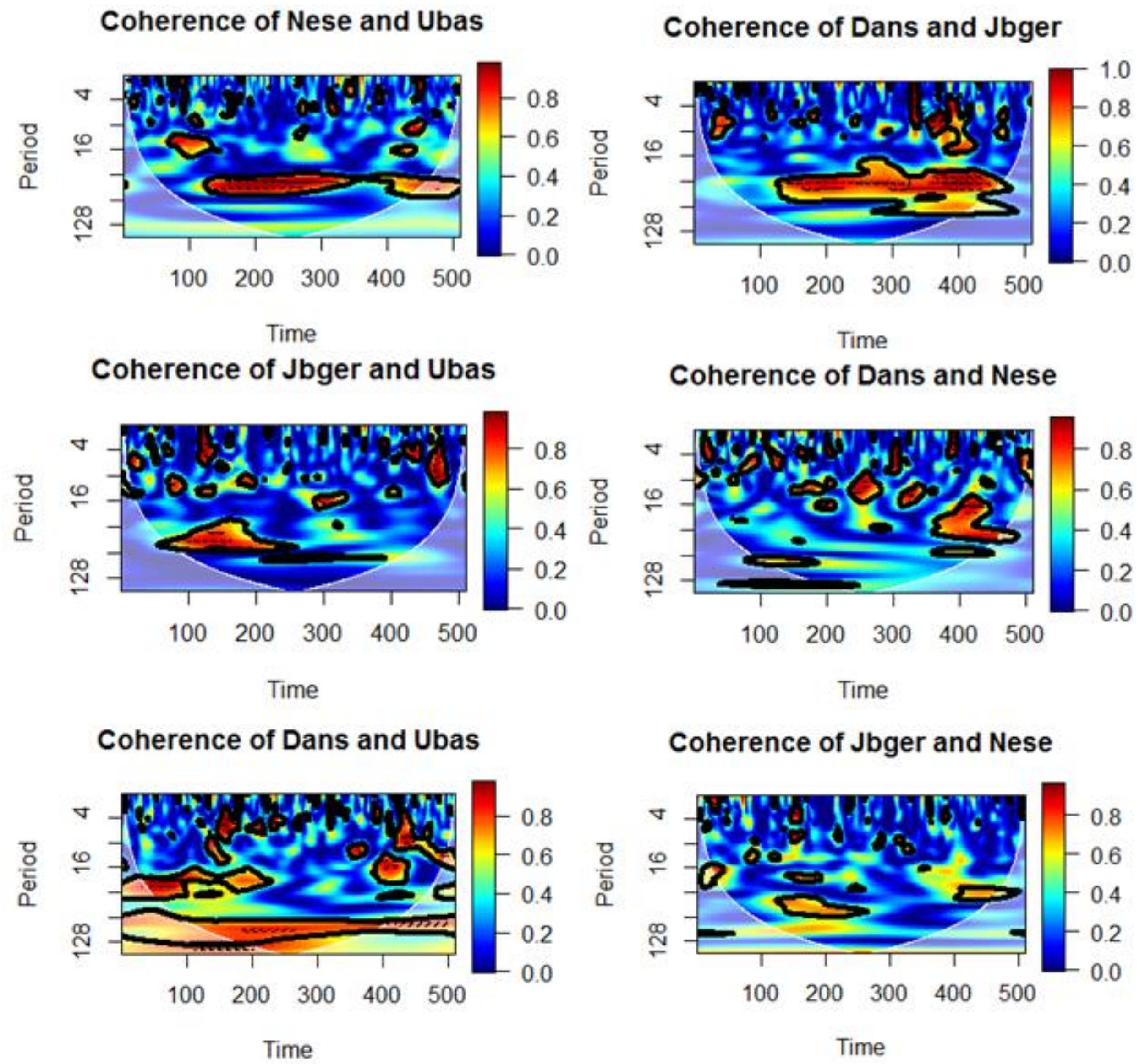


Figure 3 Wavelet Coherence

Reference

António, R., & Luis, C. N. (March 2009). International comovement of stock market returns: a wavelet analysis. Lisboa: Banco de Portugal.

Debnath, L., & Bhatta, D. (2015). Integral Transform and their Applications. New York: CRC Press.

Gourène, G. A., & Mendy, P. (2018). Oil prices and African stock markets co-movement: A time and frequency analysis. Journal of African Trade, 5, 55–67.

Grinsted, A., Moore, J. C., & Jevrejeva, S. (2004). Application of the cross wavelet transform and wavelet coherence to geophysical time series. Nonlinear Processes in Geophysics, 11, 561–566.

Jammazi, R., & Aloui, C. (2012). Crude oil price forecasting: experimental evidence from wavelet decomposition and neural network modeling. Energy Econ. , 34 (3), 828–841.

Masih, M., & Majid, H. A. (2013). Comovement of Selected International Stock Market Indices:A Continuous Wavelet Transformation and Cross Wavelet Transformation Analysis. MPRA posted 4. September 2014 (58313).

Okonkwo, C. U., Osu, B. O., & Chibuisi, C. (2018). Modelling Stochastic Volatility of The Stock Market: A Nigerian Experience. Math LAB Journal, 1 (3), 259-267.

Ramsey, J. B., & Lampart, C. (1998). The decomposition of economic relationships by time scale using wavelets: Expenditure and income.

Studies in Nonlinear Dynamics and Economics, 3, 23–42.

Razdan, A. (2004). Wavelet correlation coefficient of strongly correlated time series. Physica A: Statistical Mechanics and its Applications, 333 (0), 335 -342.

Tiwari, A. K., Dar, A. B., Bhanja, N., & Shah, A. (2013). Stock Market Integration in Asian Countries: evidence from Wavelet multiple correlations. Journal of Economic Integration, 28 (3), 441~456.

Tkacz, G. (2001). Estimating the fractional order of integration of interest rates using a wavelet OLS estimator. Studies in Nonlinear Dynamics and Econometrics, 5 (1), 1 - 14.

Vavr̃ina, M. (2012). Co-movement of Stock Markets and Commodities: A Wavelet Analysis. Charles University in Prague, Faculty of Social Sciences, Institute of Economic Studies.

Vo, X. H. (2011). A wavelet-based approach to model oil and stock market relationships. Pantheon Sorbonne.: University of Paris 1.